*J. Fluid Mech.* (1983), *vol.* 137, *pp.* 467–469
*Printed in Great Britain*


# CORRIGENDUM

I. Sedimentation in a dilute polydisperse system of interacting spheres.
Part 1. General formulae
By G. K. BATCHELOR
*Journal of Fluid Mechanics*, vol. 119, 1982, pp. 379–408.

II. Sedimentation in a dilute polydisperse system of interacting spheres.
Part 2. Numerical results
By G. K. BATCHELOR AND C. S. WEN
*Journal of Fluid Mechanics*, vol. 124, 1982, pp. 495–528.

III. Diffusion in a dilute polydisperse system of interacting spheres
By G. K. BATCHELOR
*Journal of Fluid Mechanics*, vol. 131, 1983, pp. 155–175.

There is an error in the expression (6.12) for the direct contribution to the sedimentation coefficient $S_{ij}$ due to Brownian motion in Paper I which unfortunately has consequences for Papers II and III. In the manipulations yielding (I. 6.12) from (I. 6.4) a minus sign in front of the right-hand side was lost, and the relation should read as

$$S_{ij}^{(B)} = -(\gamma\lambda^2-1)\left(\frac{1+\lambda}{2\lambda}\right)^2 \int_2^\infty \left\{\frac{A_{11}-B_{11}}{s}+\frac{1}{2}\frac{\mathrm{d}A_{11}}{\mathrm{d}s}-\frac{2(A_{12}-B_{12})}{(1+\lambda)s}-\frac{1}{1+\lambda}\frac{\mathrm{d}A_{12}}{\mathrm{d}s}\right\} \times \exp\left(\frac{-\Phi_{ij}}{kT}\right) Q(s)\, s^2\, \mathrm{d}s. \quad \text{(I. 6.12)}$$

I am grateful to my colleagues Dr E. J. Hinch and Dr J. M. Rallison who in the course of their work on the interpretation of light-scattering measurements of particle diffusion derived the expression for $S_{ij}^{(B)}$ by a different method and detected this error.

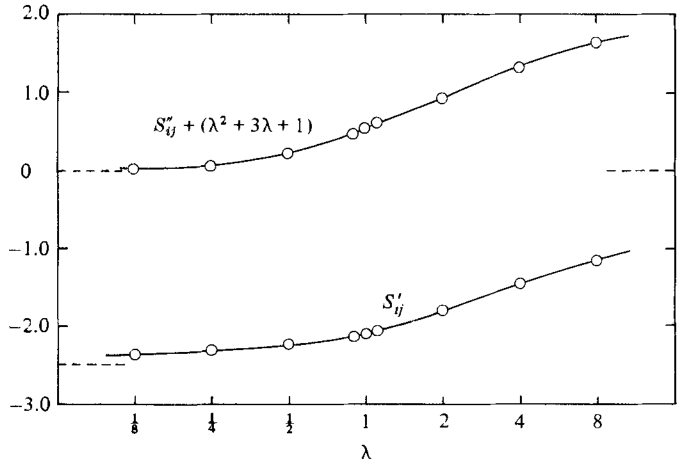
The only other correction needed to Paper I is the deletion of the clause 'which is mostly downward (or upward)' from the first sentence of the first new paragraph on page 399; this was a rash intuitive prediction, now seen to be incorrect, of the sign of the resultant of positive and negative contributions to the flux of sphere pairs due to Brownian diffusion.

A number of corrections should be made to Paper II, in which the three direct contributions to the sedimentation coefficient are evaluated numerically, and to Paper III, in which the numerical results of Paper II for small Péclet number are used again in a calculation of the particle diffusivity which is related to the sedimentation coefficient. Everywhere the correction is a change of sign of the mathematical expression for, or the numerical value of, $S_{ij}^{(B)}$ (and its constituent parts defined by the relation $S_{ij}^{(B)} = S_{ij}^{\prime(B)}+\gamma S_{ij}^{\prime\prime(B)}$), or is a direct consequence of such changes. In the following list of the places where a correction is needed, details of the change are given only where they are not evident.

## *Paper II*

Only the results for small values of the Péclet number are affected.

A minus sign should be inserted on the right-hand side of (II. 3.10), which is effectively a repetition of (I. 6.12) for the case $\Phi_{ij} = 0$. The signs of all the numerical values of $S_{ij}^{\prime(B)}$ and $S_{ij}^{\prime\prime(B)}$ in table 3 should be changed, and consequential changes are




Corrected version of figure 10 in Paper II. Calculated values of the sedimentation coefficients $S'_{ij}$ and $S''_{ij}$ as functions of $\lambda$ at small Péclet number (and $\Phi_{ij} = 0$). Both curves should asymptote to zero at sufficiently large values of $\lambda$. The bulk mobility coefficients $K'_{ij}$, $K''_{ij}$ are identical with $S'_{ij}$, $S''_{ij}$ at small Péclet number, so that this is effectively also a corrected version of figure 1 in Paper III.

needed in the values of $S'_{ij}$ $(= S'^{(G)}_{ij} + S'^{(B)}_{ij})$ and $S''_{ij}$ $(= S''^{(G)}_{ij} + S''^{(B)}_{ij})$ in later columns of the table. The values of $S'_{ij}$ and $S''_{ij}$ are also given in figure 10, of which a corrected version is shown here.

Section 4 presents calculations of the effects of an interparticle force between spheres of the same size, and since $S^{(B)}_{ij} = 0$ when $\gamma = 1$ corrections are needed only in §4.3. A minus sign is needed on the right-hand side of (II. 4.20), and the signs of all the values of $S'^{(B)}_{ij}$ given in table 6 and figure 15 should be changed. Figure 16 shows the sums of the three direct contributions to $S'_{ij}$ and to $S''_{ij}$, and the change of signs of $S'^{(B)}_{ij}$ and $S''^{(B)}_{ij}$ affects the curves in figure 16 in ways which can be determined from the corrected table 6.

In figure 17, which shows the dependence of $S_{ij}$ on $\lambda$ for the case $\gamma = 1$, the curve for the case of small Péclet number needs correction. The new values of $S_{ij}$ are the sum of the values of $S'_{ij}$ and $S''_{ij}$ given in the above revised figure 10, and are a little larger, when $\gamma > 1$, than those in figure 17. The empirical relation (II. 5.5) should be replaced by

$$S'_{ij} = \frac{-2.5}{1+0.16\lambda}, \quad S''_{ij} = \frac{\lambda^2}{1+\lambda^3} - (\lambda^2 + 3\lambda + 1), \qquad \text{(II. 5.5)}$$

which represents the calculated values of $S'_{ij}$ and $S''_{ij}$ to within 5 % over the whole range of values of $\lambda$ when $\Phi_{ij} = 0$.

## *Paper III*

The expressions for $S'^{(B)}_{ij}$ and $S''^{(B)}_{ij}$ given in (III. 3.9) are again a repetition of (I. 6.12) and need the addition of a minus sign on the right-hand side.

Table 1 and figure 1 give values of the bulk mobility coefficients $K'_{ij}$ and $K''_{ij}$, which are identical with the sedimentation coefficients $S'_{ij}$ and $S''_{ij}$ for small Péclet number, and the required changes of sign of the values of $S'^{(B)}_{ij}$ and $S''^{(B)}_{ij}$ have the same consequences as for table 3 and figure 10 in Paper II. (See the corrected version of figure 10 of Paper II above.) Likewise the empirical formula for $K'_{ij}$ in (III. 4.5) should be replaced by the new formulae in (II. 5.5) above. With these new formulae the simple expressions for the diffusivity tensor for the case $\Phi_{ij} = 0$ given in (III. 4.6, 4.7) are replaced by

$$\boldsymbol{D}_{ii} = \boldsymbol{D}_{ii}^{(0)}\left(1+1.45\phi_i - \sum_{k(\neq i)} \frac{2.5\phi_k}{1+0.16\lambda_{ik}}\right), \qquad \text{(III. 4.6)}$$

$$\underset{(i \neq j)}{\boldsymbol{D}_{ij}} = D_i^{(0)}\,\phi_i\left(\lambda^3+2\lambda^2+\frac{\lambda^2}{1+\lambda^3}\right), \qquad \text{(III. 4.7)}$$

and there are consequential changes in the expressions (III. 4.8) and (III. 4.9) for the flux of particle number and volume.

Table 2 shows the effect of an interparticle force on the values of $K'_{ij}$ and $K''_{ij}$ for the case $\lambda = 1$, and the corrections needed are the same as those to the values of $S'_{ij}$ and $S''_{ij}$ resulting from the change of sign of the contributions $S'^{(B)}_{ij}$ and $S''^{(B)}_{ij}$ $(= -S'^{(B)}_{ij}$ when $\lambda = 1)$ shown in table 6 of Paper II. Likewise the curves for $K'_{ij}$ in figure 2 need the same corrections as the curves for $S'_{ij}$ in figure 16 of Paper II.

Finally, and of most relevance to those who are interested in light-scattering measurements of particle diffusion, the estimate of the tracer diffusivity given at the bottom of page 172 needs correction. The scalar diffusivity of a tracer $i$-particle in a dispersion of $j$-particles is

$$D_i^{(0)}(1+K'_{ij}\,\phi_j),$$

and when $a_i = a_j$ and $\Phi_{ij} = 0$

$$K'_{ij} = -C+S'^{(B)}_{ij}, \qquad \text{(III. 6.3)}$$

where $C$ stands for an integral whose value is 1.83. In Paper III the value of $S'^{(B)}_{ij}$ for $\lambda = 1$ and $\Phi_{ij} = 0$ is given as 0.27, but it should be $-0.27$, so that the correct value of $K'_{ij}$ is $-2.10$. The last sentence of the paragraph ending near the top of page 173 repeats the inadequate argument for expecting a particular sign of the direct contribution to $K'_{ij}$ due to Brownian smoothing of the pair-distribution function and should be disregarded.